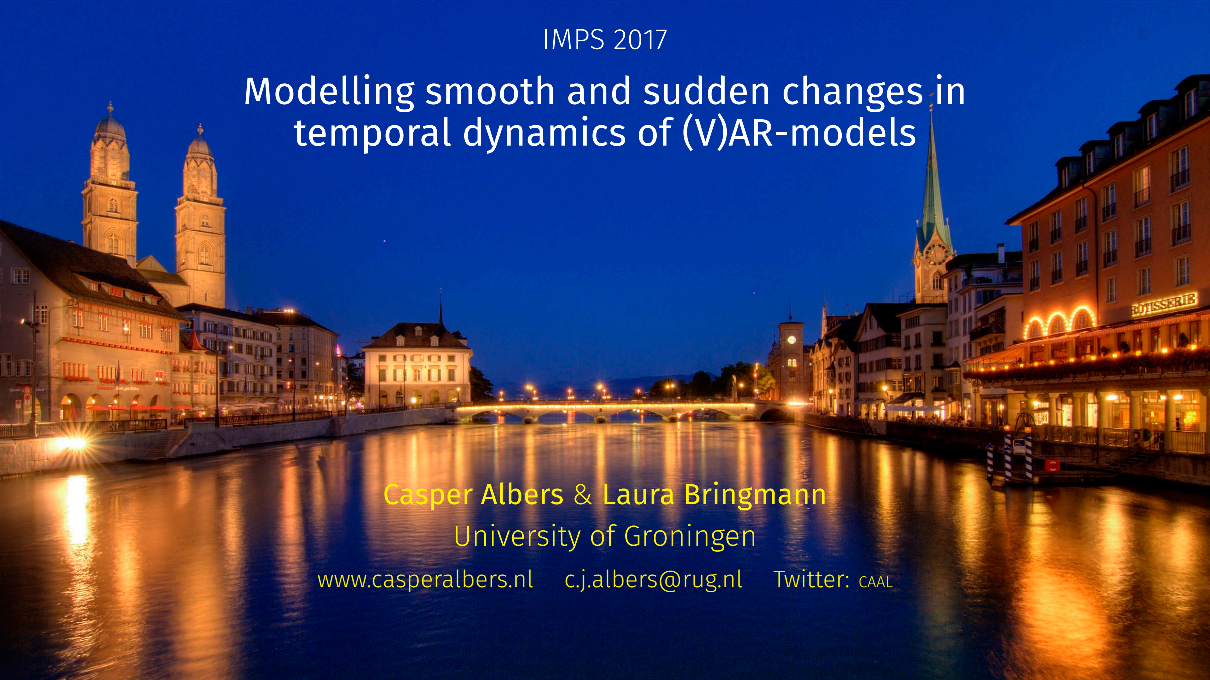

IMPS 2017

Modelling smooth and sudden changes in temporal dynamics of (V)AR-models

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The basic AR(1) model

- Series of (psychological) measurements $y_1, \ldots, y_T$.
- Simplest form of the model:

$$y_t = \mu + \phi y_{t-1} + \varepsilon_t, \quad (t = 2, \ldots, T)$$

- Assume (for now) stationarity ($|\phi| < 1$) and $\varepsilon_i \sim N(0, \sigma^2)$

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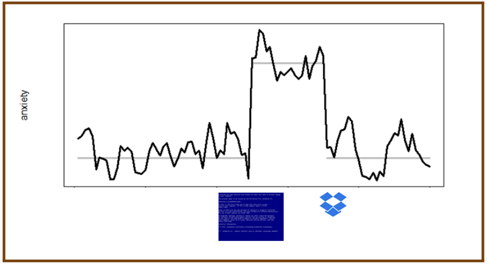
- Assume (for now) stationarity ($|\phi| < 1$) and $\varepsilon_i \sim N(0, \sigma^2)$
- Here, μ and ϕ are fixed: they can't change.
- But people do change.

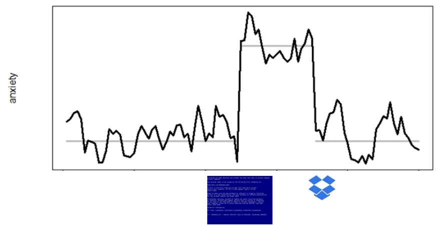
Two types of change in AR(1) models

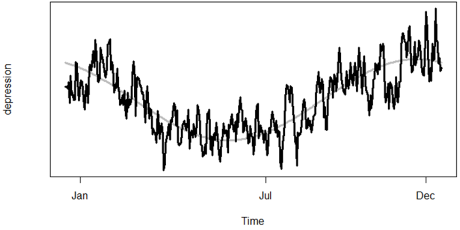
1. Sudden change



Two types of change in AR(1) models

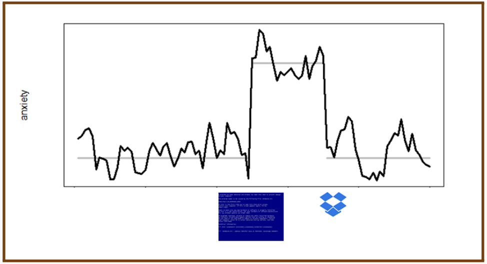
1. Sudden change
2. Smooth change





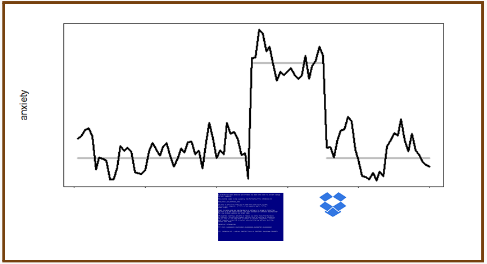
Two types of sudden change

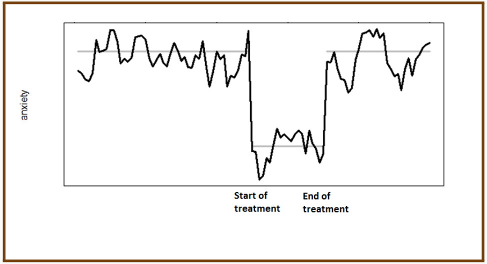
1. Sudden change at a unknown moment



Two types of sudden change

1. Sudden change at a unknown moment
2. Sudden change at an known moment





Goal of this talk

Thus, the dynamics in an AR(1) model can change

- Suddenly – at known moment(s)
- Suddenly – at unknown moment(s)
- Smoothly – (all the time)

(Many) models for one of these cases already exist. A model that combines these three cases in one is new.

My goal of the day: to introduce this model to you

Models for sudden change

Many different models exist, e.g.

- Markov switching (regime change) models (next slide)
- Models from in Statistical Quality Control (e.g. the CUSUM procedure; Page, 1954)
- Models from Deep Learning (e.g. Krylov subspace models; Ide & Tsuda, 2007)
- Models from Machine Learning (e.g. relative density-ratio method; Sugiyama, Suzuki, & Kanamori, 2012)
- . . . . (really, a *lot* of alternatives)

Regime Switching Models

- Use dummy-variable

$$D_{i,t} = \begin{cases} 0 & \text{in regime } 0 \text{ at time } t < i \\ 1 & \text{in regime } 1 \text{ at time } t \geq i \end{cases}$$

for some i.

Regime Switching Models

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  for some i.

- Then apply model

$$y_t = \mu_{D_{i,t}} + \phi y_{t-1} + \varepsilon_t, \quad (t = 2, \ldots, T) \quad \text{or}$$

$$y_t = \mu_{D_{i,t}} + \phi_{D_{i,t}}(y_t - \mu_{D_{i,t-1}}) + \varepsilon_t, \quad (t = 2, \ldots, T)$$

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  with $\mu_0 \neq \mu_1$ (Hamilton, 1989)
- Straightforward if i known. Apply HMM to find i when unknown.

Model for smooth change

For this, we use the Time-Varying Autoregressive Model (TV-AR) by Bringmann et al. (Psychological Methods, 2017).

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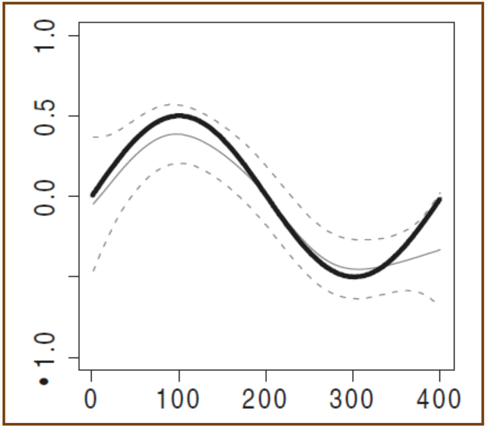
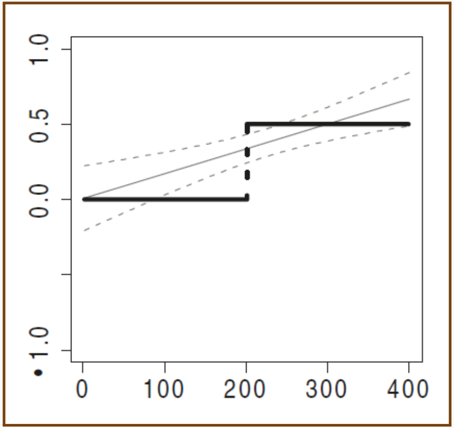
For this, we use the Time-Varying Autoregressive Model (TV-AR) by Bringmann et al. (Psychological Methods, 2017).

$$y_t = \mu_t + \phi_t y_{t-1} + \varepsilon_t$$

- μ_t and ϕ_t not fixed, yet are only allowed to vary smoothly: $\mu_t \approx \mu_{t+1}$ and $\phi_t \approx \phi_{t+1}$
- This is achieved by using Generalized Additive Models (Hastie & Tibshirani, 1990) with thin-plate splines; and the R-package *mcgv* (Wood, 2011)

TV-AR model for smooth change





works great doesn’t work

Our model – confirmatory analyses

Basic idea of our TV-AR-RS model:

Combine TV-AR's smooth parameters with Hamilton's RS idea:

$$y_t = \mu_t + \mu_{D_{i,t}} + (\phi_t + \phi_{D_{i,t}}) \times y_{t-1} + \varepsilon_t$$

(with $\mu_0 = \phi_0 = 0$)

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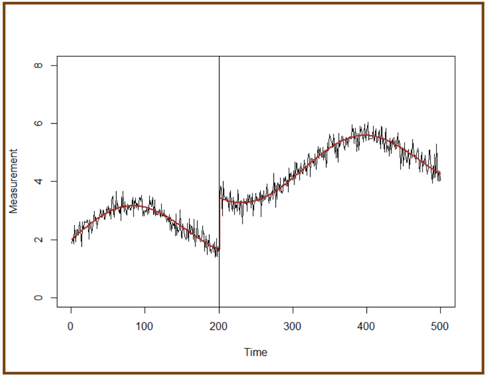
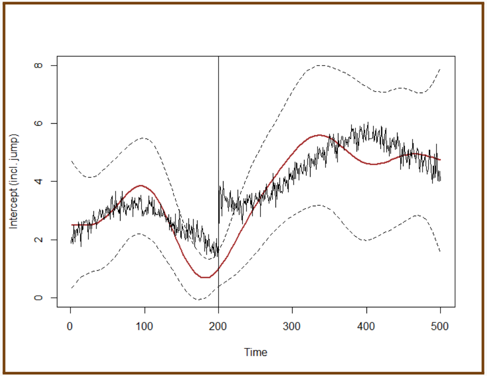
(with $\mu_0 = \phi_0 = 0$)

mcgv-package provides curves for μ_t and ϕ_t including CI, and point estimates for μ_D, ϕ_D including SE, and model fit statistics. All you need.

TV-AR-RS model – confirmatory analyses – example



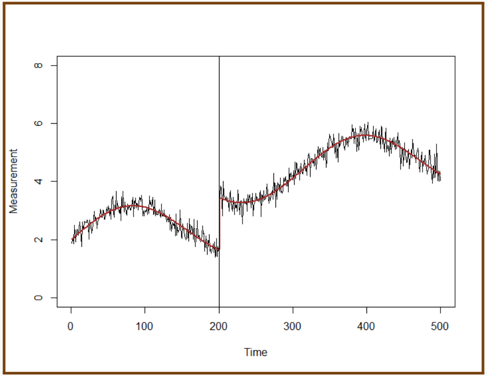
Simulated data

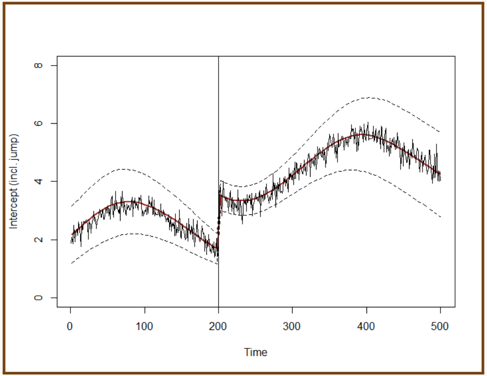


TV-AR model

TV-AR model: AIC = 173.42

TV-AR-RS model – confirmatory analyses – example





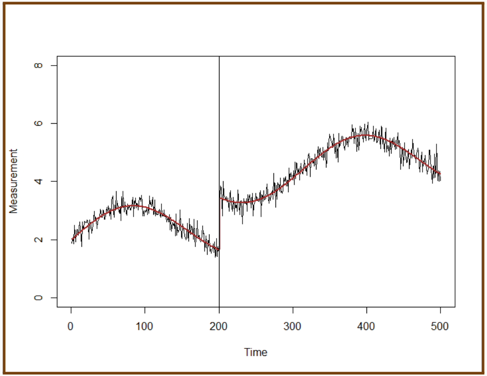
Simulated data

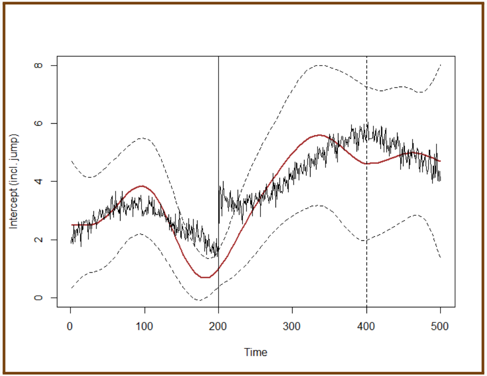
TV-AR-RS with correct jump

TV-AR model: AIC = 173.42

Correct TV-AR-RS model: AIC = 64.63, $\hat{\mu}_1 = 1.88$ (sd=.13)

TV-AR-RS model – confirmatory analyses – example





Simulated data

TV-AR-RS with correct jump

TV-AR model: AIC = 173.42

Correct TV-AR-RS model: AIC = 64.63, $\hat{\mu}_1 = 1.88$ (sd=.13)

Incorrect TV-AR-RS model: AIC = 175.27, $\hat{\mu}_1 = .04$ (sd=.13)

TV-AR-RS model – exploratory analyses

Sketch of the algorithm:

1. Compute $\mathrm{AIC}^{(0)}$ for model $y_t = \mu_t + \phi_t \times y_{t-1} + \varepsilon_t$

TV-AR-RS model – exploratory analyses

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1. Compute $\text{AIC}^{(0)}$ for model $y_t = \mu_t + \phi_t \times y_{t-1} + \varepsilon_t$
2. $\forall\, j \in \{2, \ldots, T-1\}$ compute $\text{AIC}_j^{(1)}$ for $y_t = \mu_t + \mu_{D_{j,t}} + (\phi_t + \phi_{D_{j,t}}) \times y_{t-1} + \varepsilon_t$

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3. Select $i = \mathrm{argmin}_j\ \mathrm{AIC}_j^{(1)}$
4. IF $AIC_i^{(1)} < AIC^{(0)} - 10$ THEN select point i as new change point ELSE stop

TV-AR-RS model – exploratory analyses

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3. Select $i = \text{argmin}_j\, \text{AIC}_j^{(1)}$
4. IF $AIC_i^{(1)} < AIC^{(0)} - 10$ THEN select point i as new change point ELSE stop
5. Re-run steps 2 – 4 to find subsequent change points.

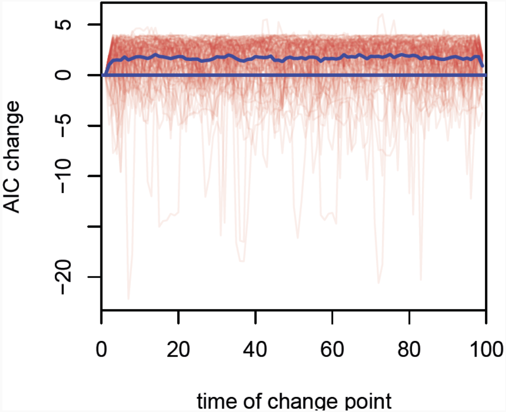
Simulation results – I

Real AR(1) model: no jumps detected

Intercept:

Autoregression:

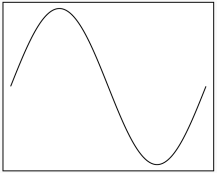
AIC change:



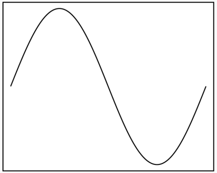
Simulation results – II

Real TV-AR(1) model: no jumps detected

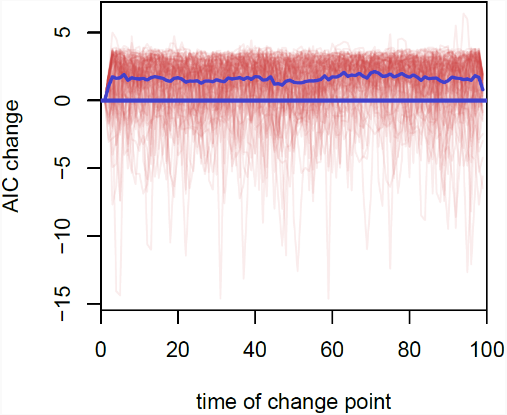
Intercept:



Autoregression:



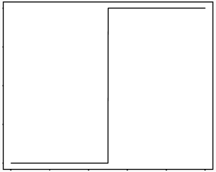
AIC change:



TV-AR-RS(1) with $\Delta\mu = 1\text{sd}$

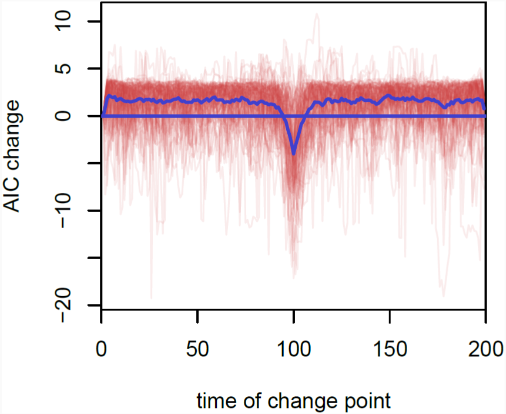
T=200 is insufficient

Intercept:



Autoregression:

AIC change:

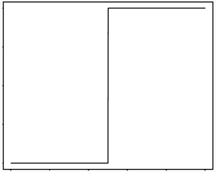


Simulation results – IV

TV-AR-RS(1) with $\Delta\mu = 2\text{sd}$

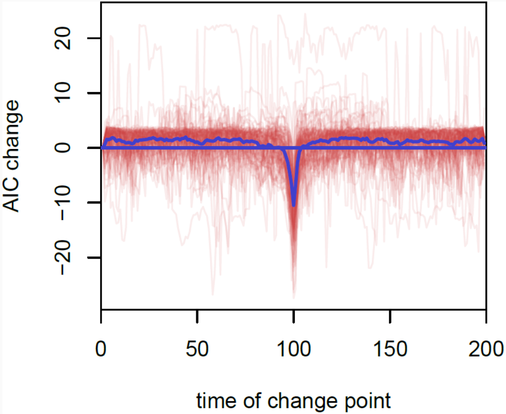
T=200 is sufficient

Intercept:



Autoregression:

AIC change:



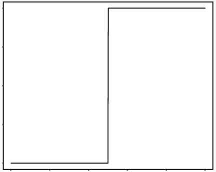
Simulation results – V

TV-AR-RS(1) with $\Delta\phi = .3$

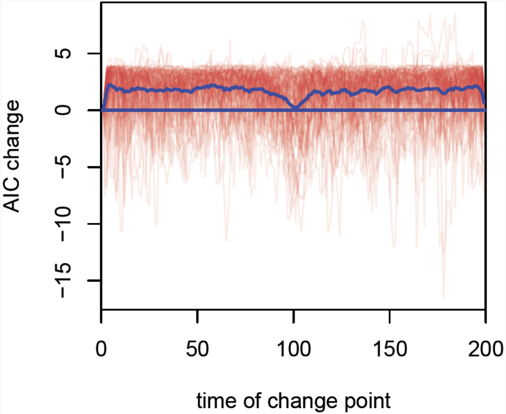
T=200 is insufficient

Intercept:

Autoregression:



AIC change:



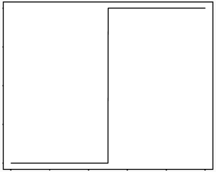
Simulation results – VI

TV-AR-RS(1) with $\Delta\phi = .7$

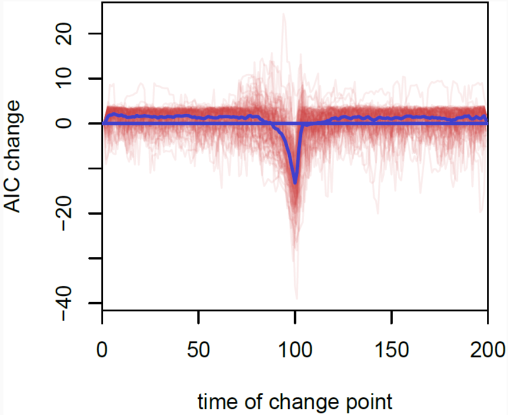
T=200 is sufficient

Intercept:

Autoregression:



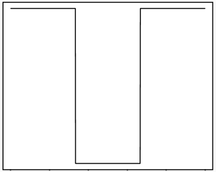
AIC change:



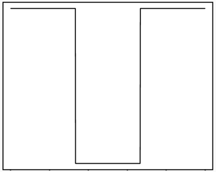
Simulation results – VII

Finding multiple jumps? Yes we can!

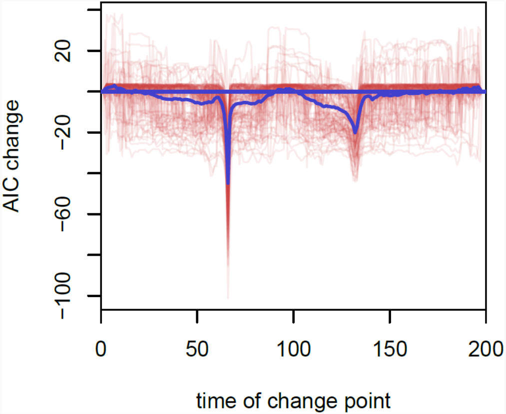
Intercept:



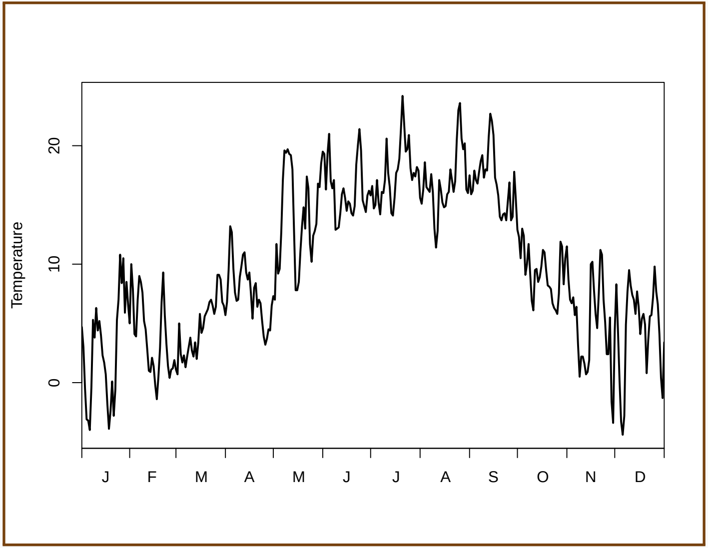
Autoregression:



AIC change:



Example: Daily temperature in Groningen/Eelde, 2016



# jumps	AIC

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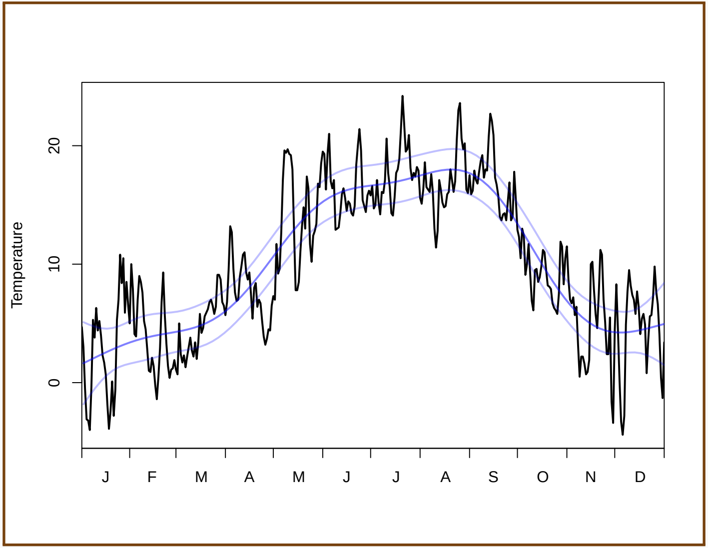
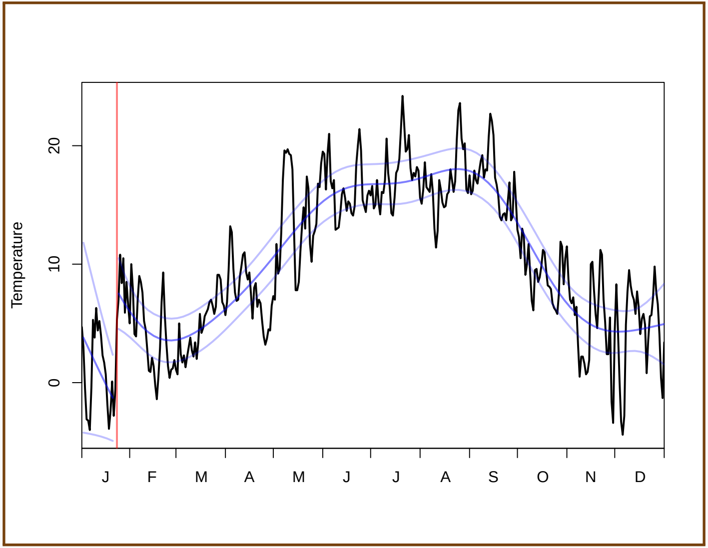


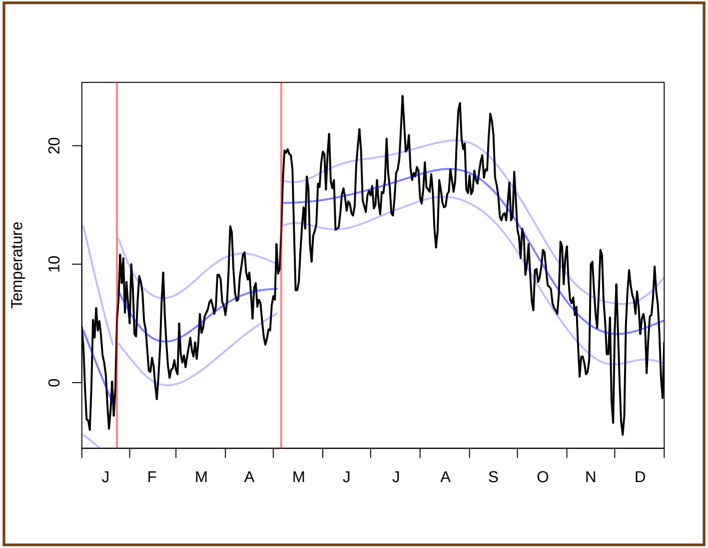
# jumps	AIC
0	1849

Example: Daily temperature in Groningen/Eelde, 2016



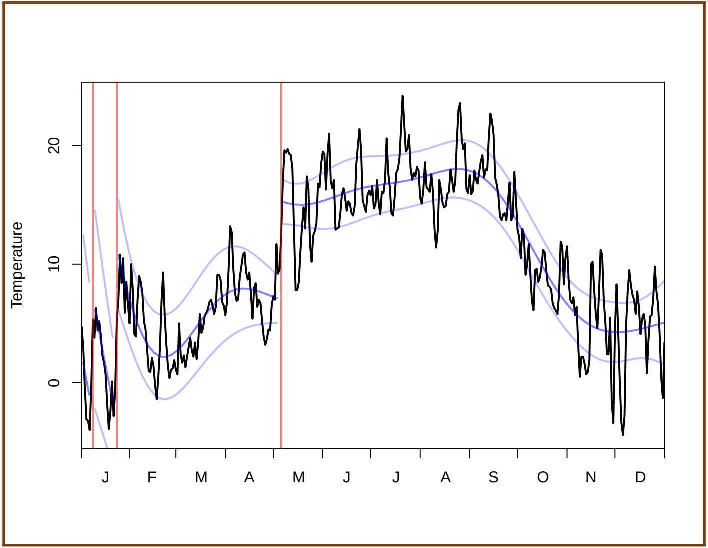
# jumps	AIC
0	1849
1	1814

Example: Daily temperature in Groningen/Eelde, 2016



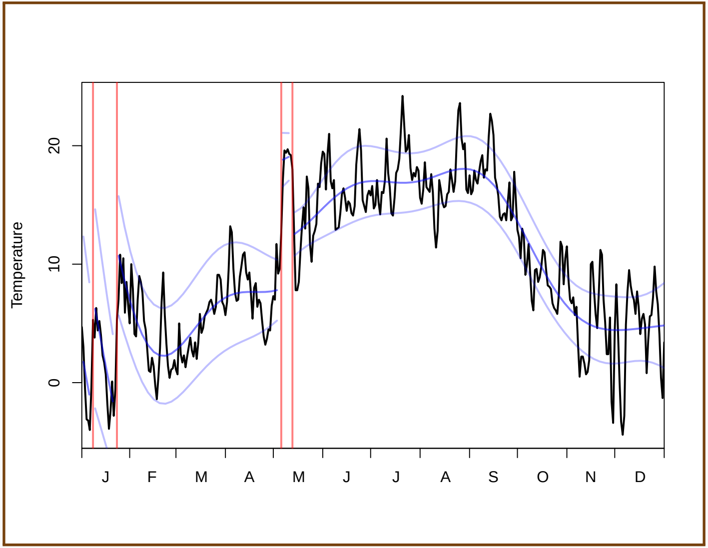
# jumps	AIC
0	1849
1	1814
2	1786

Example: Daily temperature in Groningen/Eelde, 2016



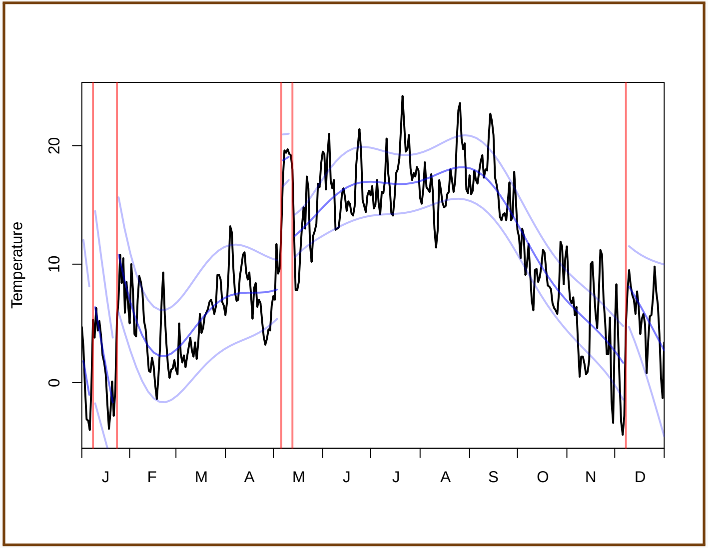
# jumps	AIC
0	1849
1	1814
2	1786
3	1747

Example: Daily temperature in Groningen/Eelde, 2016



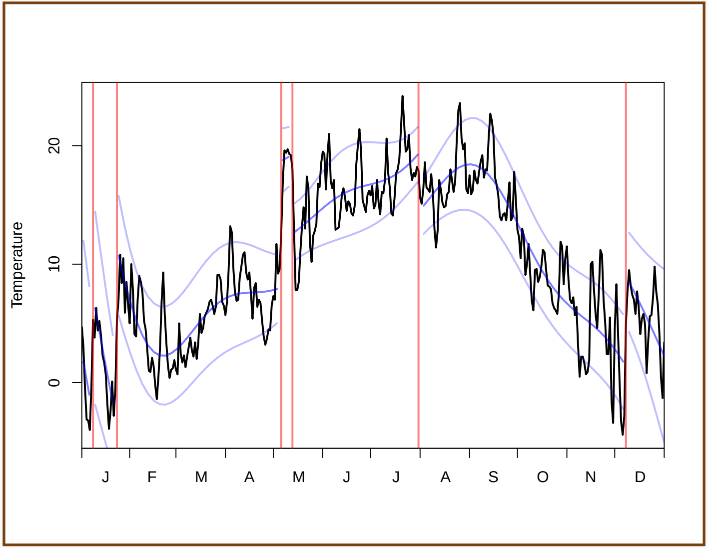
# jumps	AIC
0	1849
1	1814
2	1786
3	1747
4	1724

Example: Daily temperature in Groningen/Eelde, 2016



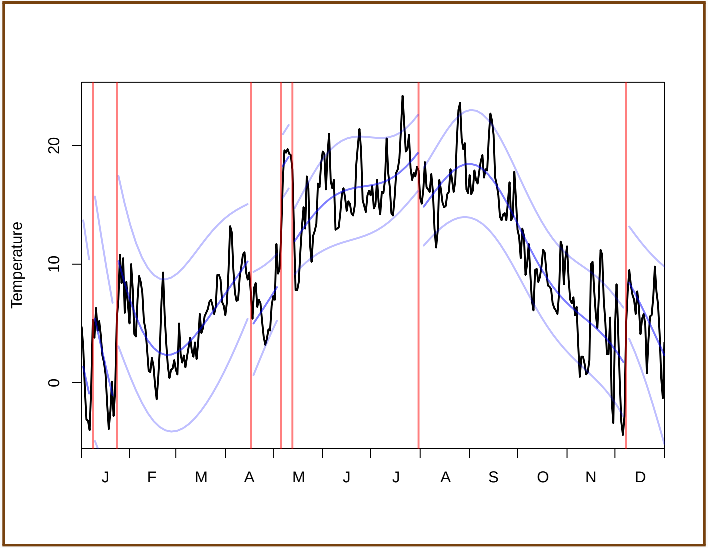
# jumps	AIC
0	1849
1	1814
2	1786
3	1747
4	1724
5	1699

Example: Daily temperature in Groningen/Eelde, 2016



# jumps	AIC
0	1849
1	1814
2	1786
3	1747
4	1724
5	1699
6	1683

Example: Daily temperature in Groningen/Eelde, 2016



# jumps	AIC
0	1849
1	1814
2	1786
3	1747
4	1724
5	1699
6	1683
7	1665

Conclusions

- We presented an elegant model that can deal with smooth and sudden change in dynamics.
- Can be used for both confirmatory and exploratory purposes
- Once finished, we will provide *R*-code
- Model works, but T needs to be large to exploratory detect sudden jumps
- Work in progress - still dotting the ι's...

Key references:

- Bringmann, Hamaker, Vigo, Aubert, Borsboom, Tuerlinckx (2017), Changing Dynamics. Psychological Methods
- Hamilton (1989). A new approach to the economic analysis of nonstationary time series and the business cycle, Econometrica
- Royal Netherlands Meteorological Institute (KNMI), www.knmi.nl, for the data.